USN

06EC65

Sixth Semester B.E. Degree Examination, Dec.2013/Jan.2014
Information Theory and Coding

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Define the following :
i) Self information ii) Entropy of the source iii) Extremal property
iv) Additive property **(04 Marks)**

b. A black and white TV picture consists of 525 lines of picture information. Assume that each line consists of 525 pixels, with each pixel having 256 equiprobable brightness levels. Pictures are repeated at the rate of r_s = 30 frames per second. Calculate the rate of information conveyed by a TV set to the viewer. **(06 Marks)**

c. The state diagram of a Markov source is shown in Fig. Q1 (c).
i) Find the state probability.
ii) Find the entropy of each state H_i.
iii) Find the entropy of the first order source H(S).
iv) Find G1, G2 and verify that, $G_1 \geq G_2 \geq H(S)$

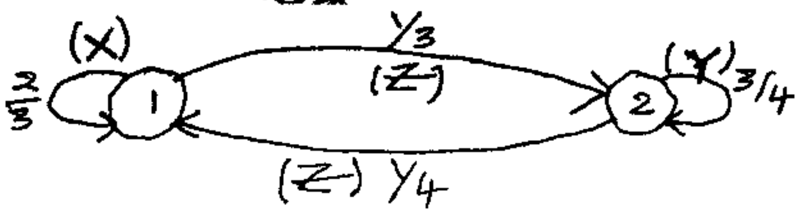

Fig. Q1 (c): Markov source for Q1 (c)

(10 Marks)

2 a. For the binary symmetric channel characterized by the following noise matrix and given symbol input probability matrix find,
i) All entropies H(X), H(Y), H(X/Y), H(Y/X), H(XY).
ii) Data transmission rate.
iii) Channel capacity given r_s = 1000 symbols / sec.
iv) Channel efficiency and redundancy.

$$P(Y/X) = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}, \; P(X) = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$ **(10 Marks)**

b. A source emits a independent sequence of symbols from an alphabet consisting of five symbols from an alphabet consisting of five symbols A, B, C, D, E with probabilities $\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{3}{16}, \frac{5}{16}$ respectively. Find the Shannon code using Shannon encoding algorithm and compute the efficiency. **(06 Marks)**

c. Prove that $H(XY) = H(Y) + H(X/Y)$ **(04 Marks)**

3 a. Design a ternary Huffman code for a source $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ with $P = \left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{12}, \frac{1}{12}\right\}$, $X = \{0, 1, 2\}$, where X is code alphabet; by moving the combined symbols i) as high as possible ii) as low as possible. For each of the codes, find
i) Minimum average length ii) Variance iii) Code efficiency. **(12 Marks)**

b. For a binary erasure channel shown in Fig. Q3 (b) find the following:
i) Average mutual information
ii) Channel capacity.
iii) Values of $P(x_1)$ and $P(x_2)$ for maximum mutual information. **(08 Marks)**

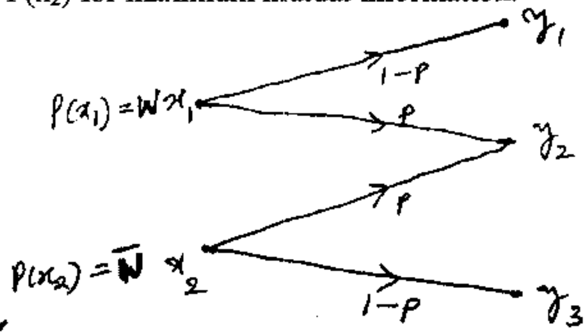

Fig. Q3 (b)

4 a. Derive an equation for the capacity of a channel of bandwidth B_{Hz} effected by additive white Gaussian noise of power spectral density η and hence prove Shannons limit $C = 1.44\, S/\eta$ bps. **(08 Marks)**

b. An analog signal has a 4 kHz bandwidth. The signal is sampled at 2.5 times the Nyquist rate and each sample quantized into 256 equally likely levels. Assume that the successive levels are statistically independent.
i) Find the information rate of the source.
ii) Can the output of this source be transmitted without errors over a Gaussian channel of bandwidth 50 kHz and $\left(\frac{S}{N}\right)$ ratio of 20 dB?
iii) If the output of this source is to be transmitted without errors over an analog channel having $\left(\frac{S}{N}\right)$ of 10 dB, compute the bandwidth requirement of the channel. **(08 Marks)**

c. Define differential entropy and briefly explain with relevant equations. **(04 Marks)**

PART – B

5 a. Explain the matrix representation of linear block codes. **(06 Marks)**

b. For a systematic linear block code, the parity matrix P is given by,

$$[P] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

i) Find all possible code vectors.
ii) Draw the corresponding encoding circuit.
iii) A single error has occurred in each of the following received vectors. Detect and correct those errors:
$R_A = [0\ 1\ 1\ 1\ 1\ 1\ 0]$, $R_B = [1\ 0\ 1\ 1\ 1\ 0\ 0]$
iv) Draw the syndrome calculation circuit. **(14 Marks)**

6 a. What is a binary cyclic code? Discuss the encoding and decoding circuit used to generate binary cyclic codes. **(06 Marks)**

b. A (15, 5) linear cyclic code has a generator polynomial,

$g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$

i) Draw the encoder and syndrome calculator circuit for this code.

ii) Find the code polynomial for the message polynomial $D(x) = 1 + x^2 + x^4$ in systematic form.

iii) Is $V(x) = 1 + x^4 + x^6 + x^8 + x^{14}$ a code polynomial? If not, find the syndrome of $V(x)$. **(14 Marks)**

7 Explain the following error control codes:

a. Golay codes.

b. Shortened cyclic codes.

c. RS codes.

d. Burst and Random error correcting codes. **(20 Marks)**

8 Consider the (3, 1, 2) convolution code with $g^{(1)} = (1\ 1\ 0)$, $g^{(2)} = (1\ 0\ 1)$ and $g^{(3)} = (1\ 1\ 1)$.

i) Draw the encoder block diagram.

ii) Find the generator matrix.

iii) Find the codeword corresponding to the information sequence 1 1 1 0 1, using the time domain and transform domain approach. **(20 Marks)**